Announcements

1) Exam next week on Wednesday, Review session Monday, Practice problems up on CTools later today.

Kirchoff's Voltage Law

- $v_1, v_2, \ldots, v_n$ electric potential (voltage) at nodes - in volts

$$A \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$ gives the potential differences

$$A \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

only when all "loop conditions" are met.

This is inside our large matrix formula (ohm's law with zero resistance)

Matrix Operations

(Section 2.1)

We have already observed linearity:

If A is an $m \times n$ matrix, x and y are n-vectors, and c is a scalar,

1) $A(x+y) = Ax + Ay$

2) $A(cx) = c(Ax)$

Example 1:

Suppose

$$Ax = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$ and

$$Ay = \begin{bmatrix} -6 \\ 52 \end{bmatrix}.$$

Find $A(3x-2y)$.

$$A(3x-2y) = A(3x) - A(2y)$$

(property 1)

$$= 3Ax - 2Ay$$

(property 2)

$$= 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} - 2\begin{bmatrix} -6 \\ 52 \end{bmatrix}$$

$$3\begin{bmatrix} 2 \\ 1 \end{bmatrix} - 2\begin{bmatrix} -6 \\ 52 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 3 \end{bmatrix} - \begin{bmatrix} -12 \\ 104 \end{bmatrix}$$

$$= \begin{bmatrix} 6+12 \\ 3-104 \end{bmatrix} = \begin{bmatrix} 18 \\ -101 \end{bmatrix}$$

A function from $\mathbb{R}^n$ to $\mathbb{R}^m$ satisfying properties 1) and 2) is called linear.

Fact: Every such function is given by a matrix.

Matrix Addition

Let $A = (a_{i,j})_{i=1, j=1}^{m \quad n}$

be an $\underline{m \times n}$ matrix whose

entry on the i^{th} row

and j^{th} column is given by

$a_{i,j}$. Let $B = (b_{i,j})_{i=1, j=1}^{m \quad n}$

be another $\underline{m \times n}$ matrix.

We define

$A+B$ to be the

$m \times n$ matrix

$$\boxed{A+B = \left(a_{i,j}+b_{i,j}\right)_{i=1,\, j=1}^{m \quad n}}$$

i.e. the entry in the i^{th} row and j^{th} column is the sum of the entries of A and B from the same row and column.

Example 2:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 5 \\ -3 & 15 & 22 \end{bmatrix}$$

(2×3)

$$B = \begin{bmatrix} 0 & -6 & -10 \\ 56 & 15 & 6 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1+0 & 2-6 & 5-10 \\ -3+56 & 15+15 & 22+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & -5 \\ 53 & 30 & 28 \end{bmatrix}$$

Matrix Multiplication

Really just function composition

Let A be an $m \times n$ matrix
$ ||$
and let B be an $n \times k$ matrix. Then B acts on k-vectors to produce an n-vector and A acts on n-vectors to produce an m-vector.

We define the matrix product AB as a function from $\mathbb{R}^k$ to $\mathbb{R}^m$ given by

$$\boxed{(AB)x = A(Bx)}$$

for any k-vector x.

This is a linear function, so it is given by a matrix.

The entries of

AB are given by

$c_{i,j}$ where $1 \leq i \leq m$,

$1 \leq j \leq k$ and

$\boxed{\text{$c_{i,j}$ is the dot product of the i^{th} row of A with the j^{th} column of B.}}$

Usually write $c_{i,j} = (AB)_{i,j}$

Example 3:

Let $A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 4 \end{bmatrix}$

(2×3)

$B = \begin{bmatrix} 0 & -1 \\ 2 & 5 \\ 1 & 0 \end{bmatrix}$

What is the entry in AB on the 2^{nd} row and first column?

This is the second row of A dotted with the first column of B, so we get

$$\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = 8$$

$AB =$

$$\begin{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} & \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix} \\ \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 8 & 11 \end{bmatrix}$$